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Inflation and Nonminimal Scalar-Curvature Coupling in Gravity and Supergravity

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Abstract

Inflationary slow-roll dynamics in Einstein gravity with a nonminimal scalar-curvature coupling can be equivalent to that in the certain $f(R)$ gravity theory. We review the correspondence and extend it to N=1 supergravity. The nonminimal coupling in supergravity is rewritten in terms of the standard ('minimal') N=1 matter-coupled supergravity by using curved superspace. The established equivalence between two different inflationary theories means *the same* inflaton scalar potential, and does not imply the same post-inflationary dynamics and reheating.

1 Introduction

The standard mechanism of inflation in field theory uses the Einstein gravity minimally coupled to a non-gravitational scalar field (called *inflaton*) whose potential energy drives inflation. The inflaton scalar potential should be flat enough to meet the slow-roll conditions during the inflationary period. This is not the only possibility since inflaton may have a purely geometrical origin and then gravity has to be modified also. However, as we argue below, both alternatives represent the particular cases of the more general situation when inflaton field is non-minimally coupled to gravity that leads to a variant of scalar-tensor gravity. Great success of inflationary cosmology over the recent past, which has led to the definitive predictions about the post-inflationary Universe confirmed by observations, has left unanswered the fundamental question about the *origin* of inflaton particle and its scalar potential. Knowing the inflaton origin would fix inflaton *interactions* with other particles. In turn, it would lead to definitive physical predictions about reheating after inflation and the origin of all known elementary particles from inflaton decay.

One may pursue two different strategies in a theoretical search for inflaton. Inflaton may be either a new exotic particle or something that we already know 'just around the corner'. We adopt the second ('economical') approach. In this paper we consider the two natural possibilities: Higgs inflation and $(R + R^2)$ inflation, where R is the Ricci scalar. In the Higgs inflation, inflaton is identified with Higgs particle nonminimally coupled to gravity [1] (see also [2, 3, 4, 5] for the study of the Standard Model loop corrections to the Higgs scalar potential and their effect on observational quantities). In the $(R+R^2)$ inflation [6, 7] (see also [8]), which historically was the first fully developed inflationary model having a graceful exit from the initial de Sitter stage to the final radiation dominated Friedmann-Robertson-Walker (FRW) stage through an intermediate period of matter creation and heating, the inflaton field (dubbed scalaron) is spin-0 part of spacetime metric, so it has the purely geometrical origin.

Because inflationary dynamics of the universe is fully determined by the effective inflaton potential, some apparently very different models of inflation may lead to the same inflationary physics. It occurs in the case under our consideration also. Indeed, it was already noticed in [2] that the spectra of scalar and tensor perturbations generated in the $(R + R^2)$-inflationary model [9] are the *same* as those in the Higgs inflationary model with the tree level Higgs potential [1] in the limit of the infinitely large coupling $\xi \to \infty$ (see also [10]). This equivalence of observational predictions is the consequence of the asymptotic duality of both models. Namely, the inflaton scalar potential, derived from the nonminimal coupling [1], does coincide with the effective inflaton scalar potential that follows

from the $(R+R^2)$-inflationary model [6, 7] in the limit $\xi \to \infty$. [1] The main topic of our paper is upgrade of this duality to N=1 supergravity.

The established duality of both models is valid during inflation and even for some time after it, but not for the whole post-inflationary evolution. Reheating mechanisms in those models are different too. As a result, the *reheating* temperature $T_{\rm reh}$ after Higgs inflation is about 10^{13} GeV [1], whereas after $(R+R^2)$ inflation one finds $T_{\rm reh} \approx 10^9$ GeV [7], see also [11].

The motivation of [1] was based on the 'most minimal' assumption that there is no new physics beyond the Standard Model up to the Planck scale. The most economical mechanism of inflation can be based on a new (beyond the Standard Model) Higgs dynamics instead of introducing a new particle. We assume the new physics beyond the Standard Model, which is given by supersymmetry. Then it is quite natural to search for the most economical mechanism of inflation in the context of supergravity. And we are not forced to identify inflaton with a Higgs particle of the Minimal Supersymmetric Standard Model.

Our paper is organized as follows. In sec. 2 we briefly review chaotic inflation with nonminimal coupling to gravity [1] (see also the old paper [12] albait without a connection to Higgs scalar). In sec. 3 we briefly review chaotic inflation in $(R+R^2)$ gravity [6, 7], and prove its duality to that of sec. 2. In sec. 4 we outline a construction of the new supergravity theory proposed in ref. [13] and called $F(\mathcal{R})$ supergravity. Our main results are given in secs. 5 and 6. Our conclusions are in sec. 7.

2 Inflation with nonminimal coupling to gravity

Consider the 4D Lagrangian

$$\mathcal{L}_{\rm J} = \sqrt{-g_{\rm J}} \left\{ -\frac{1}{2}(1+\xi\phi_{\rm J}^2)R_{\rm J} + \frac{1}{2} g_{\rm J}^{\mu\nu}\partial_\mu\phi_{\rm J}\partial_\nu\phi_{\rm J} - V(\phi_{\rm J}) \right\} \tag{1}$$

where we have introduced the real scalar field $\phi_{\rm J}(x)$, nonminimally coupled to gravity (with the coupling constant ξ) in the Jordan frame, with the Higgs-like scalar potential

$$V(\phi_{\rm J}) = \frac{\lambda}{4}(\phi_{\rm J}^2 - v^2)^2 \tag{2}$$

We use the units $\hbar = c = M_{\rm Pl} = 1$, where $M_{\rm Pl}$ is the reduced Planck mass, with the spacetime signature $(+,-,-,-)$.

[1] This duality also provides a novel microscopic mechanism of generating macroscopic $f(R)$ gravity which, in contrast to the one-loop quantum gravitational effects, does *not* lead to the appearance of the extra $R_{\mu\nu}R^{\mu\nu}$ term producing undesirable ghost or tachyon massive gravitons at the quasi-classical level. However, *not any* $f(R)$ gravity theory can be obtained this way.

The action (1) can be rewritten to Einstein frame by redefining the metric via a Weyl transformation,

$$g^{\mu\nu} = \frac{g_{\rm J}^{\mu\nu}}{1+\xi\phi_{\rm J}^2} \tag{3}$$

It gives rise to the standard Einstein-Hilbert term $(-\frac{1}{2}R)$ for gravity in the Lagrangian. However, it also leads to a nonminimal (or noncanonical) kinetic term of the scalar field $\phi_{\rm J}$. To get the canonical kinetic term, a scalar field redefinition is needed, $\phi_{\rm J} \to \varphi(\phi_{\rm J})$, subject to the condition

$$\frac{d\varphi}{d\phi_{\rm J}} = \frac{\sqrt{1+\xi(1+6\xi)\phi_{\rm J}^2}}{1+\xi\phi_{\rm J}^2} \tag{4}$$

As a result, the non-minimal theory (1) is classically equivalent to the standard (canonical) theory of the scalar field $\varphi(x)$ minimally coupled to gravity,

$$\mathcal{L}_{\rm E} = \sqrt{-g}\left\{-\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right\} \tag{5}$$

with the scalar potential

$$V(\varphi) = \frac{V(\phi_{\rm J}(\varphi))}{[1+\xi\phi_{\rm J}^2(\varphi)]^2} \tag{6}$$

Given a large positive $\xi \gg 1$, in the small field limit one finds from eq. (4) that $\phi_{\rm J} \approx \varphi$, whereas in the large φ limit one gets

$$\varphi \approx \sqrt{\frac{3}{2}}\log(1+\xi\phi_{\rm J}^2) \tag{7}$$

Then eq. (6) yields the scalar potential:
(i) in the *very small* field limit, $v \ll \varphi < \sqrt{\frac{2}{3}}\xi^{-1}$, as

$$V_{\rm vs}(\varphi) \approx \frac{\lambda}{4}\varphi^4 \tag{8}$$

(ii) in the *small* field limit, $\sqrt{\frac{2}{3}}\xi^{-1} < \varphi \ll \sqrt{\frac{3}{2}}$, as

$$V_{\rm s}(\varphi) \approx \frac{\lambda}{6\xi^2}\varphi^2, \tag{9}$$

(iii) and in the *large* field limit, $\varphi \gg \sqrt{\frac{2}{3}}\xi^{-1}$, as

$$V(\varphi) \approx \frac{\lambda}{4\xi^2}\left(1-\exp\left[-\sqrt{\frac{2}{3}}\varphi\right]\right)^2 \tag{10}$$

We have assumed here that $\xi \gg 1$ and $v\xi \ll 1$.

It was proposed in ref. [1] to identify inflaton with Higgs particle, which requires the parameter v to be the order of weak scale, and the coupling λ be the Higgs boson selfcoupling at the inflationary scale. The scalar potential (10) is well known to be perfectly suitable to support a slow-roll inflation, while its consistency with the COBE normalization condition for the observed CMB amplitude of density perturbations (eg., at the e-foldings number $N_e = 50 \div 60$) gives rise to $\xi/\sqrt{\lambda} \approx 5 \cdot 10^4$ [1]. The scalar potential (9) corresponds to the post-inflationary matter-dominated epoch described by the oscillating inflaton field φ with the frequency

$$\omega = \sqrt{\frac{\lambda}{3}}\,\xi^{-1} \tag{11}$$

3 Inflation in $(R + R^2)$ gravity

It has been known for a long time [6, 7] that viable inflationary models can be easily constructed in (non-supersymmetric) $f(R)$-gravity theories (see eg., refs. [14] for a recent review) with the action

$$S = \int d^4x\sqrt{-g}\; f(R) \tag{12}$$

whose function $f(R)$ begins with the Einstein-Hilbert term, $(-\frac{1}{2}R)$, while the rest takes the form $R^2C(R)$ for $R \to \infty$, with a slowly varying function $C(R)$. The simplest model is given by $C(R) = const. \neq 0$ with

$$f(R) = -\frac{1}{2}\left(R - \frac{R^2}{6M^2}\right) \tag{13}$$

The theory (13) is known as the excellent model of chaotic inflation [15, 9, 16]. The coefficient in front of the second term on the right-hand-side of eq. (13) is chosen so that M actually coincides with the rest mass of the scalar particle (scalaron) appearing in $f(R)$-gravity at low curvatures $|R| \ll M^2$ or in flat spacetime, in particular. The model fits the observed amplitude of scalar perturbations if $M/M_{\rm Pl} \approx 1.5 \cdot 10^{-5}(50/N_e)$, and gives rise to the spectral index $n_s - 1 \approx -2/N_e \approx -0.04(50/N_e)$ and the scalar-to-tensor ratio $r \approx 12/N_e^2 \approx 0.005(50/N_e)^2$, in terms of the e-foldings number $N_e \approx (50 \div 55)$ depending upon details of reheating after inflation [7, 11, 16]. Despite of the fact that it has been known for more than 30 years, the model (13) remains viable and is in agreement with the most recent WMAP7 observations of $n_s = 0.963 \pm 0.012$ and $r < 0.24$ (with 95% CL) [17].

As is also well known [18], any $f(R)$ gravity theory is classically equivalent to the scalar-tensor gravity with the Brans-Dicke paremeter $\omega_{\rm BD} = 0$. In order

to derive the corresponding scalar potential, one rewrites the theory (12) to the equivalent form

$$S_A = \int d^4x\sqrt{-g}\,[AR - Z(A)] \tag{14}$$

where the scalar field A has been introduced. Via eliminating the scalar field A by its algebraic equation of motion from the action (14) one gets back the original action (12) provided that the functions f and Z are related via Legendre transformation,

$$f(R) = RA(R) - Z(A(R)) \tag{15}$$

It follows, in particular, that

$$Z'(A) = R \qquad \text{and} \qquad f'(R) = A \tag{16}$$

where the primes denote the derivatives with respect to the given argument.

A Weyl transformation

$$g_{\mu\nu} \to g_{\mu\nu} \exp\left[-\sqrt{\frac{2}{3}}\varphi\right] \tag{17}$$

with the conformal factor

$$\exp\left[\sqrt{\frac{2}{3}}\varphi\right] = A \tag{18}$$

allows one to bring the action (14) to Einstein frame with the canonical kinetic terms,

$$S_\varphi = \int d^4x\,\sqrt{-g}\left\{-\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}\exp\left[\frac{-4\varphi}{\sqrt{6}}\right]Z(A(\varphi))\right\} \tag{19}$$

in terms of the physical (and canonically normalized) scalar field φ, with the scalar potential

$$V(\varphi) = -\frac{1}{2}\exp\left[\frac{-4\varphi}{\sqrt{6}}\right]Z\left(\exp\left[\sqrt{\frac{2}{3}}\varphi\right]\right) \tag{20}$$

In the special case [6, 7]

$$f_{\rm S}(R) = -\frac{1}{2}\left(R - \frac{1}{6M^2}R^2\right) \tag{21}$$

one finds

$$V(\varphi) = \frac{3}{4}M^2\left(1 - \exp\left[-\sqrt{\frac{2}{3}}\varphi\right]\right)^2 \tag{22}$$

This inflaton scalar potential is *the same* as the one in eq. (10) provided that we identify the couplings as

$$3M^2 = \frac{\lambda}{\xi^2} \tag{23}$$

Therefore, we conclude that the physical consequences for inflation in the model with the nonminimal scalar-curvature coupling (Sec. 2) and the $(R + R^2)$ model of this section are essentially the same. In particular, the inflaton mass is given by

$$M = \frac{1}{\xi}\sqrt{\frac{\lambda}{3}} \tag{24}$$

and is equal to the frequency (ω) of small metric and curvature oscillations around the FRW background solutions with low curvature.

4 Nonminimal coupling in supergravity

In 4D, N=1 supersymmetry, gravity is to be extended to N=1 supergravity, while a scalar field should be complexified and become the leading complex scalar field component of a chiral (scalar) matter supermultiplet. In a curved superspace of N=1 supergravity, the chiral matter supermultiplet is described by a covariantly chiral superfield Φ obeying the constraint $\overline{\nabla}_{\dot{\alpha}}\Phi = 0$. See the textbooks [19, 20, 21] for more details. We use here the notation of Wess and Bagger [20]. The standard (generic and minimally coupled) matter-supergravity action reads in superspace as follows:

$$S_{\rm MSG} = -3\int d^4x d^4\theta E^{-1} \exp\left[-\frac{1}{3}K(\Phi,\overline{\Phi})\right] + \left\{\int d^4x d^2\theta \mathcal{E} W(\Phi) + \text{H.c.}\right\} \tag{25}$$

in terms of the Kähler potential K and the superpotential W of the chiral supermatter, and the full density E and the chiral density $\mathcal{E}$ of the superspace supergravity. It is convenient to introduce the notation

$$\Omega = -3\exp\left[-\frac{1}{3}K\right] \quad \text{or} \quad K = -3\ln\left[-\frac{1}{3}\Omega\right] \tag{26}$$

The non-minimal matter-supergravity coupling in superspace reads

$$S_{\rm NM} = \int d^4x d^2\theta \mathcal{E} X(\Phi)\mathcal{R} + \text{H.c.} \tag{27}$$

in terms of the chiral function $X(\Phi)$ and the N=1 chiral scalar supercurvature superfield $\mathcal{R}$ obeying $\overline{\nabla}_{\dot{\alpha}}\mathcal{R} = 0$. In terms of the field components of the superfields the non-minimal action (27) is given by

$$\int d^4x d^2\theta \mathcal{E} X(\Phi)\mathcal{R} + \text{H.c.} = -\frac{1}{6}\int d^4x\sqrt{-g}X(\phi_c)R + \text{H.c.} + \ldots \tag{28}$$

where the dots stand for the fermionic terms, and $\phi_c = \Phi| = \phi + i\chi$ is the leading complex scalar field component of the superfield Φ. Given $X(\Phi) = -\xi\Phi^2$ with the real coupling constant ξ, we find the bosonic contribution

$$S_{\rm NM,bos.} = \frac{1}{6}\xi \int d^4x\sqrt{-g}\left(\phi^2 - \chi^2\right) R \tag{29}$$

It is worth noticing that the supersymmetrizable (bosonic) non-minimal coupling reads $\left[\phi_c^2 + (\phi_c^\dagger)^2\right] R$, not $(\phi_c^\dagger\phi_c)R$.

Let's introduce the manifestly supersymmetric nonminimal action (in Jordan frame) as

$$S = S_{\rm MSG} + S_{\rm NM} \tag{30}$$

In curved superspace of N=1 supergravity the (Siegel's) chiral integration rule

$$\int d^4x d^2\theta \mathcal{E}\mathcal{L}_{\rm ch} = \int d^4x d^4\theta E^{-1}\frac{\mathcal{L}_{\rm ch}}{\mathcal{R}} \tag{31}$$

applies to any chiral superfield Lagrangian $\mathcal{L}_{\rm ch}$ with $\overline{\nabla}_{\dot{\alpha}}\mathcal{L}_{\rm ch} = 0$. It is, therefore, possible to rewrite eq. (30) to the equivalent form

$$S_{\rm NM} = \int d^4x d^4\theta E^{-1}\left[X(\Phi) + \overline{X}(\overline{\Phi})\right] \tag{32}$$

We conclude that adding $S_{\rm NM}$ to $S_{\rm MSG}$ is equivalent to the simple change of the Ω-potential as (*cf.* ref. [22])

$$\Omega \to \Omega_{\rm NM} = \Omega + X(\Phi) + \bar{X}(\overline{\Phi}) \tag{33}$$

Because of eq. (26), it amounts to the change of the Kähler potential as

$$K_{\rm NM} = -3\ln\left[e^{-K/3} - \frac{X(\Phi) + \overline{X}(\overline{\Phi})}{3}\right] \tag{34}$$

The scalar potential in the matter-coupled supergravity (25) is given by [23]

$$V(\phi,\bar{\phi}) = e^G\left[\left(\frac{\partial^2 G}{\partial\phi\partial\bar{\phi}}\right)^{-1}\frac{\partial G}{\partial\phi}\frac{\partial G}{\partial\bar{\phi}} - 3\right] \tag{35}$$

in terms of the *single* (Kähler-gauge-invariant) function

$$G = K + \ln|W|^2 \tag{36}$$

Hence, in the nonminimal case (30) we have

$$G_{\rm NM} = K_{\rm NM} + \ln|W|^2 \tag{37}$$

Contrary to the bosonic case, one gets a nontrivial Kähler potential $K_{\rm NM}$, ie. a *Non-Linear Sigma-Model* (NLSM) as the kinetic term of $\phi_c = \phi + i\chi$ (see eg., ref. [24] for more about the NLSM). Since the NLSM target space in general has a nonvanishing curvature, no field redefinition generically exist that could bring the kinetic term to the free (canonical) form with its Kähler potential $K_{\rm free} = \overline{\Phi}\Phi$.

5 $F(\mathcal{R})$ supergravity and chaotic inflation

The textbook (Poincaré) supergravity [19, 20, 21] is the N=1 locally supersymmetric extension of Einstein gravity, with the Einstein-Hilbert gravitational term in the action. The manifestly N=1 locally supersymmetric extension of all $f(R)$ gravity theories (12) was constructed in curved superspace only recently [13] and was called $F(\mathcal{R})$ supergravity. [2]

The $F(\mathcal{R})$ supergravity is nicely formulated in a chiral 4D, N=1 superspace where it is defined by the action

$$S = \int d^4x d^2\theta\, \mathcal{E} F(\mathcal{R}) + \text{H.c.} \tag{38}$$

in terms of a holomorphic function $F(\mathcal{R})$ of the covariantly-chiral scalar curvature superfield $\mathcal{R}$, and the chiral superspace density $\mathcal{E}$. The chiral $N = 1$ superfield $\mathcal{R}$ has the scalar curvature R as the field coefficient at its θ^2-term. The chiral superspace density $\mathcal{E}$ (in a WZ gauge) reads

$$\mathcal{E} = e\left(1 - 2i\theta\sigma_a\bar{\psi}^a + \theta^2 B\right) \tag{39}$$

where $e = \sqrt{-g}$, ψ^a is gravitino, and $B = S - iP$ is the complex scalar auxiliary field (it does not propagate in the theory (38) despite of the apparent presence of the higher derivatives). The full component structure of the action (38) is very complicated. Nevertheless, it is classically equivalent to the standard N=1 Poincaré supergravity minimally coupled to the chiral scalar superfield, via the supersymmetric Legendre-Weyl-Kähler transform [13]. The chiral scalar superfield is the superconformal mode of a supervielbein (in Minkowski or AdS vacuum).

A relation to the $f(R)$-gravity theories is established by dropping the gravitino ($\psi^a = 0$) and restricting the auxiliary field B to its real (scalar) component, $B = 3X$ with $\overline{X} = X$. Then, as is shown in ref. [13], the bosonic Lagrangian takes the form

$$L = 2F'\left[\tfrac{1}{3}R + 4X^2\right] + 6XF \tag{40}$$

It follows that the auxiliary field X obeys an algebraic equation of motion,

$$3F + 11F'X + F''\left[\tfrac{1}{3}R + 4X^2\right] = 0 \tag{41}$$

In those equations $F = F(X)$ and the primes denote the derivatives with respect to X. Solving eq. (41) for X and substituting the solution back into eq. (40) results in the bosonic function $f(R)$. The physical sector of the $F(\mathcal{R})$ supergravity is larger than that of the usual supergravity (ie. graviton and gravitino) due to the extra scalar (inflaton), its pseudo-scalar superpartner (axion) and inflatino.

[2] A component field construction, by the use of the 4D, N=1 superconformal tensor calculus, was proposed in ref. [25].

It is natural to expand the input function $F(\mathcal{R})$ in power series of $\mathcal{R}$. For instance, when $F(\mathcal{R}) = f_0 - \frac{1}{2} f_1 \mathcal{R}$ with some (non-vanishing and complex) coefficients f_0 and f_1, one recovers the standard *pure* N=1 Poincaré supergravity with a negative cosmological term [13, 26]. A generic $\mathcal{R}^2$ supergravity and the corresponding $f(R)$ functions were studied in ref. [26]. The most relevant term for the slow-roll chaotic inflation in $F(\mathcal{R})$ supergravity is *cubic* in $\mathcal{R}$. In refs. [27, 28] we examined the model with

$$F(\mathcal{R}) = f_0 - \tfrac{1}{2} f_1 \mathcal{R} + \tfrac{1}{2} f_2 \mathcal{R}^2 - \tfrac{1}{6} f_3 \mathcal{R}^3 \tag{42}$$

whose real coupling constants $f_{0,1,2,3}$ are of (mass) dimension 3, 2, 1 and 0, respectively. The stability conditions (ie. the absence of ghost and tachyonic degrees of freedom) require $f_1 > 0$ and $f_3 > 0$, whereas the stability of the bosonic embedding in $F(\mathcal{R})$ supergravity requires $F'(X) < 0$ [26, 27]. For the choice (42) the last condition implies

$$f_2^2 < f_1 f_3 \tag{43}$$

In ref. [27] we used the stronger conditions

$$f_3 \gg 1 \ , \qquad f_2^2 \gg f_1 \qquad \text{and} \qquad f_2^2 \ll f_1 f_3 \tag{44}$$

The first condition above is needed to have inflation at the curvatures much less than $M_{\rm Pl}^2$ (and to meet observations), while the second condition is needed to have the scalaron (inflaton) mass be much less than $M_{\rm Pl}$, in order to avoid large (gravitational) quantum loop corrections after the end of inflation up to the present time. The last condition in eq. (44) was used in ref. [27] for simplicity: then the second term on the right-hand-side of eq. (42) does not affect inflation.

Equation (40) with the Ansatz (42) in the case of $f_0 = 0$ (for simplicity) reads

$$L = -5f_3 X^4 + 11 f_2 X^3 - (7f_1 + \tfrac{1}{3} f_3 R) X^2 + \tfrac{2}{3} f_2 R X - \tfrac{1}{3} f_1 R \tag{45}$$

and gives rise to a cubic equation on X,

$$X^3 - \left(\frac{33 f_2}{20 f_3}\right) X^2 + \left(\frac{7 f_1}{10 f_3} + \frac{1}{30} R\right) X - \frac{f_2}{30 f_3} R = 0 \tag{46}$$

The high curvature regime including inflation is given by

$$\delta R < 0 \quad \text{and} \quad \frac{|\delta R|}{R_0} \gg \left(\frac{f_2^2}{f_1 f_3}\right)^{1/3} \tag{47}$$

where we have introduced the notation $R_0 = 21 f_1/f_3 > 0$ and $\delta R = R + R_0$. With our sign conventions we have $R < 0$ during the de Sitter and matter dominated stages. In the regime (47) the f_2-dependent terms in eqs. (45) and (46) can be neglected, and we get

$$X^2 = -\tfrac{1}{30} \delta R \tag{48}$$

and

$$L = -\frac{f_1}{3}R + \frac{f_3}{180}(R+R_0)^2 \tag{49}$$

The value of the coefficient R_0 is not important in the high-curvature regime. In fact, it may be changed by the constant $f_0 \neq 0$ in the Ansatz (42). Thus eq. (49) reproduces the inflationary model (13) since inflation occurs at $|R| \gg R_0$. Therefore, we can identify

$$f_3 = \frac{15}{M^2} \approx 6 \cdot 10^{10} \tag{50}$$

where we have used the WMAP-fixed inflaton mass M value [27].

The only significant difference with respect to the original $(R+R^2)$ inflationary model is the scalaron mass that becomes much larger than M in supergravity, soon after the end of inflation when δR becomes positive. However, it only makes the scalaron decay faster and creation of the usual matter (reheating) more effective.

The whole series in powers of $\mathcal{R}$ may also be considered, instead of the limited Ansatz (42). The only necessary condition for embedding inflation is that f_3 should be anomalously large. When the curvature grows, the $\mathcal{R}^3$-term should become important much earlier than the convergence radius of the whole series without that term.

The model (42) with a sufficiently small f_2 obeying the conditions (44) gives a simple (economic) and viable realization of the chaotic $(R+R^2)$-type inflation in supergravity, thus overcoming the known (generic) difficulty with realization of chaotic inflation in supergravity known as the η-problem [29], without resorting to flat directions and without an extra matter superfield with the tuned superpotential needed for stabilization of the inflationary trajectory [30, 31, 32].

6 Chaotic inflation in $F(\mathcal{R})$ supergravity and non-minimal coupling

Let's now consider the full action (30) under the slow-roll condition, ie. when the contribution of the kinetic term is negligible. Then eq. (30) takes the truly chiral form

$$S_{\rm ch.} = \int d^4x d^2\theta \mathcal{E} \left[X(\Phi)\mathcal{R} + W(\Phi) \right] + \text{H.c.} \tag{51}$$

When choosing X as the independent chiral superfield, $S_{\rm ch.}$ can be rewritten to the form

$$S_{\rm ch.} = \int d^4x d^2\theta \mathcal{E} \left[X\mathcal{R} - \mathcal{Z}(X) \right] + \text{H.c.} \tag{52}$$

where we have introduced the notation

$$\mathcal{Z}(X) = -W(\Phi(X)) \tag{53}$$

In its turn, the action (52) is equivalent to the chiral $F(\mathcal{R})$ supergravity action (38), whose function F is related to the function $\mathcal{Z}$ via Legendre transformation,

$$\mathcal{Z} = X\mathcal{R} - F \ , \quad F'(\mathcal{R}) = X \quad \text{and} \quad \mathcal{Z}'(X) = \mathcal{R} \tag{54}$$

It implies the equivalence between the reduced action (51) and the corresponding $F(\mathcal{R})$ supergravity whose F-function obeys eq. (54).

Next, let us consider the special case of eq. (51) when the superpotential is given by

$$W(\Phi) = \frac{1}{2}m\Phi^2 + \frac{1}{6}\tilde{\lambda}\Phi^3 \tag{55}$$

with the real coupling constants $m > 0$ and $\tilde{\lambda} > 0$. The model (55) is known as the *Wess-Zumino* (WZ) model in 4D, N=1 rigid supersymmetry. It has the most general renormalizable scalar superpotential in the absence of supergravity. In terms of the field components, it gives rise to the Higgs-like scalar potential.

For more simplicity, let's take a cubic superpotential,

$$W_3(\Phi) = \frac{1}{6}\tilde{\lambda}\Phi^3 \tag{56}$$

or just assume that this term dominates in the superpotential (55), and choose the $X(\Phi)$-function in eq. (51) in the form

$$X(\Phi) = -\xi\Phi^2 \tag{57}$$

with a large positive coefficient ξ, $\xi > 0$ and $\xi \gg 1$, in accordance with eq. (28).

Let's also simplify the F-function of eq. (42) by keeping only the most relevant cubic term,

$$F_3(\mathcal{R}) = -\frac{1}{6}f_3\mathcal{R}^3 \tag{58}$$

It is straightforward to calculate the $\mathcal{Z}$-function for the F-function (58) by using eq. (54). We find

$$-X = \frac{1}{2}f_3\mathcal{R}^2 \quad \text{and} \quad \mathcal{Z}'(X) = \sqrt{\frac{-2X}{f_3}} \tag{59}$$

Integrating the last equation with respect to X yields

$$\mathcal{Z}(X) = -\frac{2}{3}\sqrt{\frac{2}{f_3}}(-X)^{3/2} = -\frac{2\sqrt{2}}{3}\frac{\xi^{3/2}}{f_3^{1/2}}\Phi^3 \tag{60}$$

where we have used eq. (57). In accordance to eq. (53), the $F(\mathcal{R})$-supergravity $\mathcal{Z}$-potential (60) implies the superpotential

$$W_{\rm KS}(\Phi) = \frac{2\sqrt{2}}{3}\frac{\xi^{3/2}}{f_3^{1/2}}\Phi^3 \tag{61}$$

It coincides with the superpotential (56) of the WZ-model, provided that we identify the couplings as

$$f_3 = \frac{32\xi^3}{\tilde{\lambda}^2} \tag{62}$$

We thus conclude that the original nonminimally coupled matter-supergravity theory (30) in the slow-roll approximation with the superpotential (56) is classically equivalent to the $F(\mathcal{R})$-supergravity theory with the F-function given by eq. (58) when the couplings are related by eq. (62).

The inflaton mass M in the supersymmetric case, according to eqs. (50) and (62) is thus given by

$$M^2 = \frac{15\tilde{\lambda}^2}{32\xi^3} \tag{63}$$

Therefore, according to eqs. (23), (50), (62) and (63), the value of ξ in the supersymmetric case is $\xi^3_{\rm susy} = (45/32)\xi^2_{\rm bos}$, or $\xi_{\rm susy} \approx 10^3$. We have assumed here that $\tilde{\lambda} \approx \mathcal{O}(1)$.

7 Conclusion

In this paper we proved the equivalence of two different (viable and well known) physical theories of inflation (Higgs inflation vs. $(R+R^2)$ inflation), and extended it to supergravity. These results are important because resolution between various inflationary theories is one of the main objectives in modern cosmology. It follows from our results that the CMB alone cannot distinguish between the Higgs and the $(R+R^2)$ inflation. However, reheating in both theories is different. In both models gravity is modified, so the effective gravitational constant becomes time- and (generically) space-dependent, which is of particular importance during inflation. However, the physical nature of inflaton in the $f(R)$ gravity and the scalar-tensor gravity is very different. In the $f(R)$ gravity the inflaton field is the spin-0 part of metric, whereas in the scalar-tensor gravity inflaton is a matter particle. The inflaton interactions with *other* matter fields are, therefore, different in both theories. It gives rise to different inflaton decay rates and different reheating, ie. implies different physics after inflation (see ref. [10] too). The same remarks apply to the supergravity case. Nevertheless, we expect the equivalence between the non-minimally coupled supergravity and the $F(\mathcal{R})$ supergravity to hold during *initial* reheating with harmonic oscillations. In the bosonic case the equivalence holds until the inflaton field value is higher than $M_{\rm Pl}/\xi_{\rm bos} \approx 10^{-5}M_{\rm Pl}$. In the supersymmetric case we find the similar bound at $M_{\rm Pl}/\xi^{3/2}_{\rm susy} \approx 10^{-5}M_{\rm Pl}$.

It should also be emphasized that our supergravity extension of Higgs inflation is truly minimal: it does not have extra symmetries, extra fields and/or extra interactions beyond those required by $N=1$ local (Poincaré) supersymmetry (compare e.g., with the $N=1$ superconformal approach in ref. [33]).

The established equivalence also begs for a fundamental reason. In the high-curvature (inflationary) regime the R^2-term dominates over the R-term in the Starobinsky $f(R)$-gravity function (13), while the coupling constant in front of the R^2-action (12) is dimensionless. The Higgs slow-roll inflation is based on the Lagrangian (1), where the $\xi\phi_J^2$ dominates over 1 (in fact, over $M^2_{\rm Pl}$) in front of the gravitational R-term, and the relevant scalar potential is given by $V_4 = \frac{1}{4}\lambda\phi_J^4$ since the parameter v is irrelevant for inflation, while the coupling constants ξ and λ are also dimensionless. Therefore, both actions are *scale invariant* in the high field (or high energy) limit. Inflation spontaneously breaks that symmetry.

The supersymmetric case is similar: the nonminimal action (51) with the X-function (57) and the superpotential (56) also has only dimensionless coupling constants ξ and $\tilde{\lambda}$, while the same it true for the $F(\mathcal{R})$-supergravity action with the F-function (58), whose coupling constant f_3 is dimensionless too. Therefore, those actions are also asymptotically scale invariant, while inflation spontaneously breaks that invariance.

A spontaneous breaking of the scale invariance necessarily leads to Goldstone particle (dilaton) associated with the spontaneously broken scale transformations (dilatations). So, perhaps, the scalaron (inflaton) of Sec. 3 should be *identified* with the Goldstone dilaton related to the spontaneously broken scale invariance (dilatations)!

The basic field theory model, describing both inflation *and* subsequent reheating, reads (see eg., eq. (6) in the second paper of ref. [34])

$$
\begin{aligned}
L/\sqrt{-g} = {}& \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\tilde{\xi}R\chi^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m_\psi)\psi \\
& - \frac{1}{2}g^2\phi^2\chi^2 - h(\bar{\psi}\psi)\phi
\end{aligned}
\tag{64}
$$

with the inflaton scalar field ϕ interacting with another scalar field χ and a spinor field ψ. The nonminimal supergravity (30) with the WZ superpotential (55) can be considered as the N=1 locally supersymmetric extension of the basic model (64), after rescaling ϕ_c to $(1/\sqrt{2})\phi_c$ and identifying $\tilde{\xi} = -\frac{1}{3}\xi$ because of eq. (29). Therefore, *pre-heating* (ie. the nonperturbative enhancement of particle production due to broad parametric resonance [34]) is a generic feature of supergravity.

The axion χ and fermion ψ are both required by supersymmetry, being in the same chiral supermultiplet with the inflaton ϕ. The scalar interactions are

$$
V_{\rm int}(\phi,\chi) = m\hat{\lambda}\phi(\phi^2+\chi^2) + \frac{\hat{\lambda}^2}{4}(\phi^2+\chi^2)^2 \tag{65}
$$

whereas the Yukawa couplings are given by

$$
L_{\rm Yu} = \frac{1}{2}\hat{\lambda}\phi(\bar{\psi}\psi) + \frac{1}{2}\hat{\lambda}\chi(\bar{\psi}i\gamma_5\psi) \tag{66}
$$

Supersymmetry implies the unification of couplings since $h = -\frac{1}{2}\hat{\lambda}$ and $g^2 = \hat{\lambda}^2$ in terms of the single coupling constant $\hat{\lambda}$. If supersymmetry is unbroken, the masses of ϕ, χ and ψ are all the same. However, inflation already breaks supersymmetry, so the spontaneously broken supersymmetry is appropriate here.

Finally, it can be argued that the classical equivalence is *broken* in quantum theory because the classical equivalence is achieved via a non-trivial field redefinition. When doing that field redefinition in the path integrals defining those quantum theories (under their unitarity bounds), it gives rise to a non-trivial Jacobian that already implies the *quantum nonequivalence*, even before taking into account renormalization. [3]

In the supergravity case, there is one more clear reason for the quantum nonequivalence between the $F(\mathcal{R})$ supergravity and the classically equivalent matter (nonminimally) coupled supergravity. The Kähler potential of the inflaton chiral superfield is described by a *full* superspace integral and, therefore, receives quantum corrections that can easily spoil classical solutions describing an accelerating universe. Actually, it was the main reason for introducing flat directions in the Kähler potential and realizing slow-roll inflation in supergravity by the use of a chiral scalar superpotential along the flat directions [29, 30]. The $F(\mathcal{R})$ supergravity action is truly *chiral*, so that the function $F(\mathcal{R})$ is already protected against the quantum perturbative corrections given by full superspace integrals. It explains why we consider $F(\mathcal{R})$ supergravity as the viable and self-consistent alternative to the Kähler flat directions for realizing slow-roll inflation in supergravity.

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[3] See ref. [35] for the first steps of quantization with a higher time derivative.

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